SAT Math Must-Know Facts & Formulas

Numbers, Sequences, Factors

Integers:	. . . , -3, -2, -1, 0, 1, 2, 3, . . .
Rationals:	fractions, that is, anything expressable as a ratio of integers
Reals:	integers plus rationals plus special numbers such as $\sqrt{2}$, $\sqrt{3}$ and π

Order Of Operations: PEMDAS
(Parentheses / Exponents / Multiply / Divide / Add / Subtract)

Arithmetic Sequences: each term is equal to the previous term plus d

Sequence: $t_1, t_1 + d, t_1 + 2d, \ldots$

Example: $d = 4$ and $t_1 = 3$ gives the sequence 3, 7, 11, 15, . . .

Geometric Sequences: each term is equal to the previous term times r

Sequence: $t_1, t_1 \cdot r, t_1 \cdot r^2, \ldots$

Example: $r = 2$ and $t_1 = 3$ gives the sequence 3, 6, 12, 24, . . .

Factors: the factors of a number divide into that number without a remainder

Example: the factors of 52 are 1, 2, 4, 13, 26, and 52

Multiples: the multiples of a number are divisible by that number without a remainder

Example: the positive multiples of 20 are 20, 40, 60, 80, . . .

Percents: use the following formula to find part, whole, or percent

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Example: 75% of 300 is what?
Solve $x = (75/100) \times 300$ to get 225

Example: 45 is what percent of 60?
Solve $45 = (x/100) \times 60$ to get 75%

Example: 30 is 20% of what?
Solve $30 = (20/100) \times x$ to get 150

Averages, Counting, Statistics, Probability

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{sum} = \text{average} \times (\text{number of terms})$$

$$\text{mode} = \text{value in the list that appears most often}$$

$$\text{median} = \text{middle value in the list (which } \textit{must} \text{ be sorted)}$$

Example: median of $\{3, 10, 9, 27, 50\} = 10$
Example: median of $\{3, 9, 10, 27\} = (9 + 10)/2 = 9.5$

Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in $N \times M$ ways.

Probability:

$$\text{probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

Example: each SAT math multiple choice question has five possible answers, one of which is the correct answer. If you guess the answer to a question completely at random, your probability of getting it right is $1/5 = 20\%$.

The probability of two different events A and B *both* happening is $P(A \text{ and } B) = P(A) \cdot P(B)$, as long as the events are independent (not mutually exclusive).

Powers, Exponents, Roots

$$x^a \cdot x^b = x^{a+b} \qquad x^a / x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

SAT Math Must-Know Facts & Formulas

Factoring, Solving

$$(x+a)(x+b) = x^2 + (b+a)x + ab \qquad \text{"FOIL"}$$

$$a^2 - b^2 = (a+b)(a-b) \qquad \text{"Difference Of Squares"}$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$
$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

To solve a quadratic such as $x^2+bx+c=0$, first factor the left side to get $(x+a_1)(x+a_2)=0$, then set each part in parentheses equal to zero. E.g., $x^2+4x+3=(x+3)(x+1)=0$ so that $x=-3$ or $x=-1$.

To solve two linear equations in x and y: use the first equation to substitute for a variable in the second. E.g., suppose $x+y=3$ and $4x-y=2$. The first equation gives $y=3-x$, so the second equation becomes $4x-(3-x)=2 \Rightarrow 5x-3=2 \Rightarrow x=1, y=2$.

Functions

A function is a rule to go from one number (x) to another number (y), usually written

$$y = f(x).$$

For any given value of x, there can only be one corresponding value y. If $y=kx$ for some number k (example: $f(x)=0.5\cdot x$), then y is said to be *directly proportional* to x. If $y=k/x$ (example: $f(x)=5/x$), then y is said to be *inversely proportional* to x.

Absolute value:

$$|x| = \begin{cases} +x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

Lines (Linear Functions)

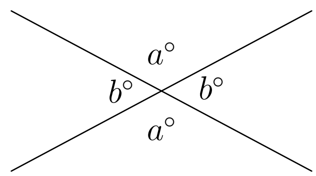
Consider the line that goes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Distance from A to B: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

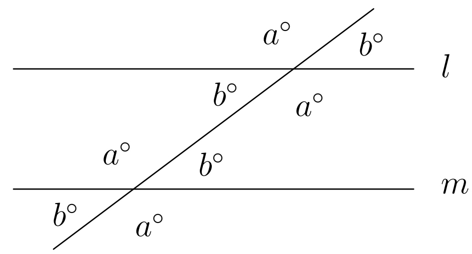
Mid-point of the segment $\overline{AB}$: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope of the line: $\frac{y_2-y_1}{x_2-x_1} = \frac{\text{rise}}{\text{run}}$

Slope-intercept form: given the slope m and the y-intercept b, then the equation of the line is $y = mx + b$. Parallel lines have equal slopes: $m_1 = m_2$. Perpendicular lines have negative reciprocal slopes: $m_1 \cdot m_2 = -1$.



Intersecting Lines



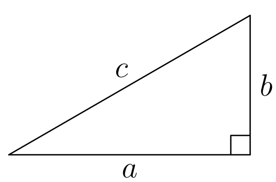
Parallel Lines ($l \parallel m$)

Intersecting lines: opposite angles are equal. Also, each pair of angles along the same line add to 180°. In the figure above, $a + b = 180°$.

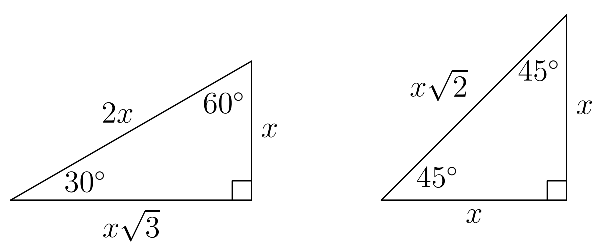
Parallel lines: eight angles are formed when a line crosses two parallel lines. The four big angles (a) are equal, and the four small angles (b) are equal.

Triangles

Right triangles:



$a^2 + b^2 = c^2$



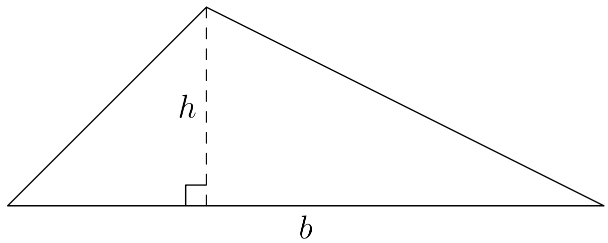
Special Right Triangles

Note that the above special triangle figures are given in the test booklet, so you don't have to memorize them, but you should be familiar with what they mean, especially the first one, which is called the Pythagorean Theorem ($a^2 + b^2 = c^2$).

A good example of a right triangle is one with $a = 3$, $b = 4$, and $c = 5$, also called a 3–4–5 right triangle. Note that multiples of these numbers are also right triangles. For example, if you multiply these numbers by 2, you get $a = 6$, $b = 8$, and $c = 10$ (6–8–10), which is also a right triangle.

The "Special Right Triangles" are needed less often than the Pythagorean Theorem. Here, "x" is used to mean any positive number, such as 1, 1/2, etc. A typical example on the test: you are given a triangle with sides 2, 1, and $\sqrt{3}$ and are asked for the angle opposite the $\sqrt{3}$. The figure shows that this angle is 60°.

All triangles:



$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

The area formula above works for *all* triangles, not just right triangles.

Angles on the inside of any triangle add up to 180°.

The length of one side of any triangle is always *less* than the sum of the lengths of the other two sides.

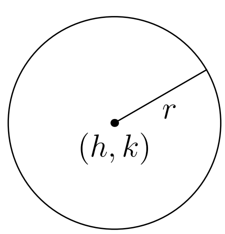
Other important triangles:

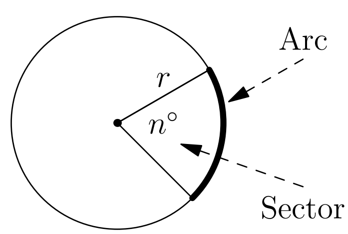
Equilateral: These triangles have three equal sides, and all three angles are 60°.

Isosceles: An isosceles triangle has two equal sides. The "base" angles (the ones opposite the two sides) are equal. A good example of an isosceles triangle is the one on page 4 with base angles of 45°.

Similar: Two or more triangles are similar if they have the same shape. The corresponding angles are equal, and the corresponding sides are in proportion. For example, the 3–4–5 triangle and the 6–8–10 triangle from before are similar since their sides are in a ratio of 2 to 1.

Circles





$\text{Area} = \pi r^2$

$\text{Circumference} = 2\pi r$

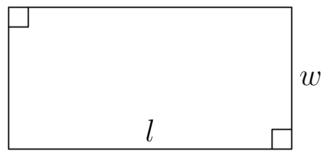
$\text{Full circle} = 360°$

(Optional)

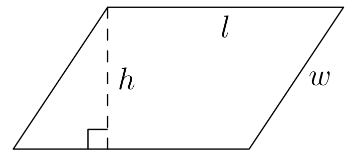
$\text{Length Of Arc} = (n°/360°) \cdot 2\pi r$

$\text{Area Of Sector} = (n°/360°) \cdot \pi r^2$

Rectangles And Friends



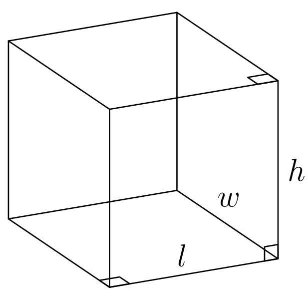
Rectangle
(Square if $l = w$)
Area $= lw$



Parallelogram (Optional)
(Rhombus if $l = w$)
Area $= lh$

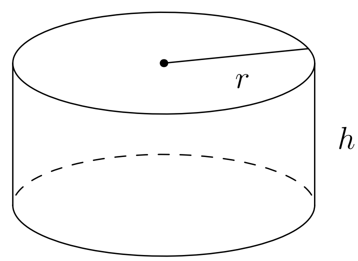
The formula for the area of a rectangle is given in the test booklet, but it is very important to know, so you should memorize it anyway.

Solids



Rectangular Solid

Volume $= lwh$



Right Cylinder

Volume $= \pi r^2 h$

Note that the above solids figures are given in the test booklet, so you don't have to memorize them, but you should be familiar with what they mean.

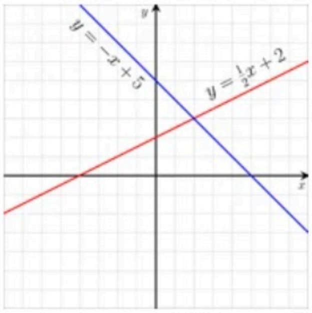
helping me!”

Formulas Not Given on the Test

For most of the formulas on this list, you'll simply need to buckle down and memorize them (sorry). Some of them, however, can be useful to know but are ultimately unnecessary to memorize, as their results can be calculated via other means. (It's still useful to know these, though, so treat them seriously).

We've broken the list into **"Need to Know"** and **"Good to Know,"** depending on if you are a formula-loving test taker or a fewer-formulas-the-better kind of test taker.

Slopes and Graphs



Need to Know

- **Slope formula**
  - Given two points, $A(x_1, y_1), B(x_2, y_2)$, find the slope of the line that connects them:

$$\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

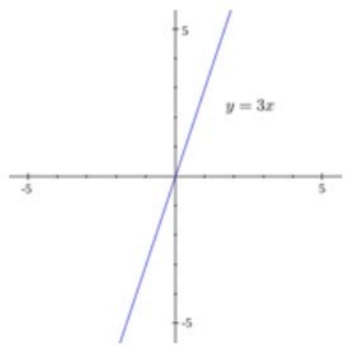
  - The slope of a line is the $\frac{\text{rise(vertical change)}}{\text{run(horizontal change)}}$.

help

- **How to write the equation of a line**
  - The equation of a line is written as:

$$y = mx + b$$

    - **If you get an equation that is NOT in this form (ex. $mx - y = b$), then re-write it into this format!** It is very common for the SAT to give you an equation in a different form and then ask you about whether the slope and intercept are positive or negative. If you don't re-write the equation into $y = mx + b$, and incorrectly interpret what the slope or intercept is, you will get this question wrong.
  - *m* is the slope of the line.
  - *b* is the y-intercept (the point where the line hits the y-axis).
  - If the line passes through the origin $(0,0)$, the line is written as $y = mx$.



Good to Know

- **Midpoint formula**
  - Given two points, $A(x_1, y_1)$, $B(x_2, y_2)$, find the midpoint of the line that connects them:

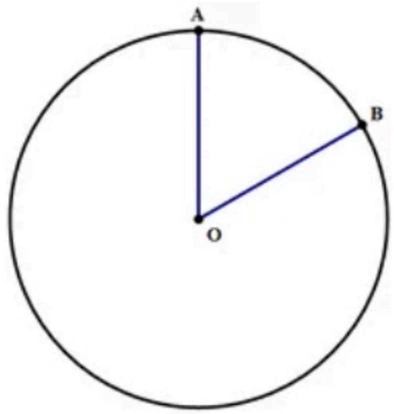
$$\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

- **Distance formula**
  - Given two points, $A(x_1, y_1), B(x_2, y_2)$, find the distance between them:

$$\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

You don't need this formula, as you can simply graph your points and then create a right triangle from them. The distance will be the hypotenuse, which you can find via the Pythagorean Theorem.

Circles



Good to Know

- **Length of an arc**
  - Given a radius and a degree measure of an arc from the center, find the length of the arc
  - Use the formula for the circumference multiplied by the angle of the arc divided by the total angle measure of the circle (360)
    - $$L_{\text{arc}} = (2\pi r)\left(\frac{\text{degree measure center of arc}}{360}\right)$$
    - E.g., A 60 degree arc is $\frac{1}{6}$ of the total circumference because $\frac{60}{360} = \frac{1}{6}$

- **Area of an arc sector**
  - Given a radius and a degree measure of an arc from the center, find the area of the arc sector
    - Use the formula for the area multiplied by the angle of the arc divided by the total angle measure of the circle
      - $$A_{\text{arc sector}} = (\pi r^2)\left(\frac{\text{degree measure center of arc}}{360}\right)$$

- **An alternative to memorizing the "formula"** is just to stop and think about arc circumferences and arc areas logically.
  - You know the formulas for the area and circumference of a circle (because they are in your given equation box on the test).
  - You know how many degrees are in a circle (because it is in your given equation box on the text).
  - Now put the two together:
    - If the arc spans 90 degrees of the circle, it must be $\frac{1}{4}$th the total area/circumference of the circle because $\frac{360}{90} = 4$. If the arc is at a 45 degree angle, then it is $\frac{1}{8}$th the circle, because $\frac{360}{45} = 8$.
    - The concept is exactly the same as the formula, but it may help you to think of it this way instead of as a "formula" to memorize.

Algebra

Need to Know

- **Quadratic equation**
  - Given a polynomial in the form of $ax^2 + bx + c$, solve for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Simply plug the numbers in and solve for x!
  - Some of the polynomials you'll come across on the SAT are easy to factor (e.g. $x^2 + 3x + 2$, $4x^2 - 1$, $x^2 - 5x + 6$, etc), but some of them will be more difficult to factor and be near-impossible to get with simple trial-and-error mental math. In these cases, the quadratic equation is your friend.
  - Make sure you don't forget to do two different equations for each polynomial: one that's $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and one that's $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

Note: If you know how to complete the square, then you don't need to memorize the quadratic equation. However, if you're not completely comfortable with completing the square, then it's relatively easy to memorize the quadratic formula and have it ready. I recommend memorizing it to the tune of either "Pop Goes the Weasel" or "Row, Row, Row Your Boat".

Averages

Need to Know

- The average is the same thing as the mean
- Find the average/mean of a set of numbers/terms

$$\text{Mean} = \frac{\text{sum of the terms}}{\text{number of different terms}}$$

- Find the average speed

$$\text{Speed} = \frac{\text{total distance}}{\text{total time}}$$

Probabilities

Need to Know

- Probability is a representation of the odds of something happening.

$$\text{Probability of an outcome} = \frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}}$$

Good to Know

- A probability of 1 is guaranteed to happen. A probability of 0 will never happen.

Percentages

Need to Know

- Find x percent of a given number n.

$$n(\frac{x}{100})$$

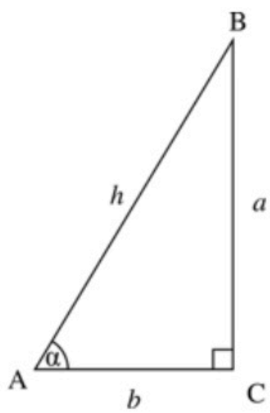
- Find out what percent a number n is of another number m.

$$\frac{(n100)}{m}$$

- Find out what number n is x percent of.

$$\frac{(n100)}{x}$$

Trigonometry



Trigonometry is a new addition to the new 2016 SAT math section. Though it makes up less than 5% of math questions, you won't be able to answer the trigonometry questions without knowing the following formulas.

Need to Know

- Find the sine of an angle given the measures of the sides of the triangle.

$sin(x)$= Measure of the opposite side to the angle / Measure of the hypotenuse

In the figure above, the sine of the labeled angle would be $\frac{a}{h}$.

- Find the cosine of an angle given the measures of the sides of the triangle.

$cos(x)$= Measure of the adjacent side to the angle / Measure of the hypotenuse

In the figure above, the cosine of the labeled angle would be $\frac{b}{h}$.

- Find the tangent of an angle given the measures of the sides of the triangle.

$tan(x)$= Measure of the opposite side to the angle / Measure of the adjacent side to the angle

In the figure above, the tangent of the labeled angle would be $\frac{a}{b}$.

- A helpful memory trick is an acronym: SOHCAHTOA.

Sine equals **O**pposite over **H**ypotenuse

Cosine equals **A**djacent over **H**ypotenuse

Tangent equals **O**pposite over **A**djacent